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JUL 77 O M PANCHENKO

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AUTHOR(S): (10) O. M. Panchenko O. M.

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AN APPROXIMATE METHOD OF CALCULATING DOWNWASH BEHIND A STRAIGHT WING WITH UNSTEADY APERIODIC MOTION AT SUBSONIC FLIGHT SPEEDS

O. M. Panchenko

Information about downwash angles behind a wing of the specified type and the character of their relationship to the angle of attack of the wing during steady and unsteady motion is vital for solving problems of the efficient configuration of the tail unit.

The calculation of downwash at the aircraft tail unit is a comparatively simple problem under stationary conditions [1]. Unsteady downwashes created by a wing are more difficult to determine.

The investigation of downwash created by a wing during harmonic vibration was presented in a quite general way in a work [2]. The question of investigating unsteady downwashes during aperiodic motion is of equal practical interest because one must study the motion of an object with aperiodic relationships of the kinematic parameters and time when the aircraft vertically zoom-climbs (breaks away), changes from one angle of attack to another, etc.

This article presents a numerical method of calculating downwash angles at the aircraft tail unit for unsteady aperiodic motion at subsonic speeds. Only straight, untwisted wings are examined, but it seems possible to devise a similar method for wings of any shape with aerodynamic or geometric twist.

A coupled system of rectangular, rectilinear coordinates xyz is introduced in which the x axis runs backward along the medial aerodynamic chord of the wing, the z axis is to the right, along the wing span on a line perpendicular to the plane of symmetry of the wing, and the y axis is above, perpendicular to the xz plane. The origin of the coordinates is at $\frac{1}{4}$ CAX (Figure 1, a). Furthermore, we shall use a relative point-grid reference, having taken half the wing span as the typical dimension:

$$\xi=\frac{x}{\frac{l}{2}};\quad \eta=\frac{y}{\frac{l}{2}};\quad \zeta=\frac{z}{\frac{l}{2}}.$$

We replace the straight wing with a system of bound vortices with axes parallel to the z axis. These vortices traverse the pressure center, whose location we assume to be $\frac{1}{4}$ CAX; their combined circulation is equal to true circulation in each wing section. A sheet of free vortices whose axes are parallel to the velocity of the free-stream flow if viewed from above streams away from the wing. This sheet is unstable and turns a certain distance behind the wing into two vortex cores (Figure 1, a) as the result of the mutual effect of the vortices that form it. One can assume that the process of turning is complete in the vicinity of the tail unit and that the vortex cores begin immediately adjacent to the wing [1, 3]. Such an arbitrary vortex model significantly simplifies solving the problem and is only valid at an adequate distance from the wing, which exists in the case of tail unit configuration on the aircraft. /7

If the vortex circulation does not change in time, then the given vortex model corresponds to steady motion. In this case the distance between the free vortices depends on the distribution law of circulation along the wing span and is calculated on the basis of the N. Ye. Zhukovskiy theorem, according to which lifts determined near the wing and far behind the wing where the effect of the bound vortices can be ignored are identical:

$$Y = 2\rho V \int_0^{\frac{l}{2}} \Gamma dz = \rho V \Gamma_0 l_1, \tag{1.1}$$

whence

$$l_1 = \frac{2\int_0^{\frac{l}{2}} \Gamma dz}{\Gamma_0},$$

where l is wing span; /8

$l_1 = \nu l$ is the distance between free vortices;

$\Gamma = f(z)$ is the law of distribution of velocity circulation along the wing span;

Γ_0 is velocity circulation in the plane of symmetry of the wing.

The law of circulation distribution along the span of a straight wing is assumed to be the average between trapezoids (with respect to the base lengths, equal to k - a value which is the reciprocal of the (wing) taper ratio and an ellipse:

$$\Gamma = 0.5\Gamma_0 [\sqrt{1-\zeta^2} + 1 - (1-k)\zeta], \tag{1.2}$$

$k = 1$ - for a parallel wing;

$k = 0$ - for a delta wing.

According to the calculations [1], the accepted law of circulation distribution coincides with the true law with sufficient accuracy. One can express circulation in the plane of symmetry of a wing Γ_0 through the lift coefficient and the geometrical characteristics of the wing by integrating the circulation curve:

$$\frac{c_y SV}{2} = 2\int_0^{\frac{l}{2}} \Gamma dz = 0{,}5\Gamma_0 l\left(\frac{\pi}{4} + \frac{1}{2} + \frac{k}{2}\right) = 0{,}5\Gamma_0 l\,(1{,}285 + 0{,}5k), \tag{1.3}$$

whence

$$\Gamma_0 = \frac{c_y SV}{l}\,\frac{1}{(1{,}285 + 0{,}5k)} = \frac{c_y lV}{\lambda}\,\frac{1}{(1{,}285 + 0{,}5k)}. \tag{1.4}$$

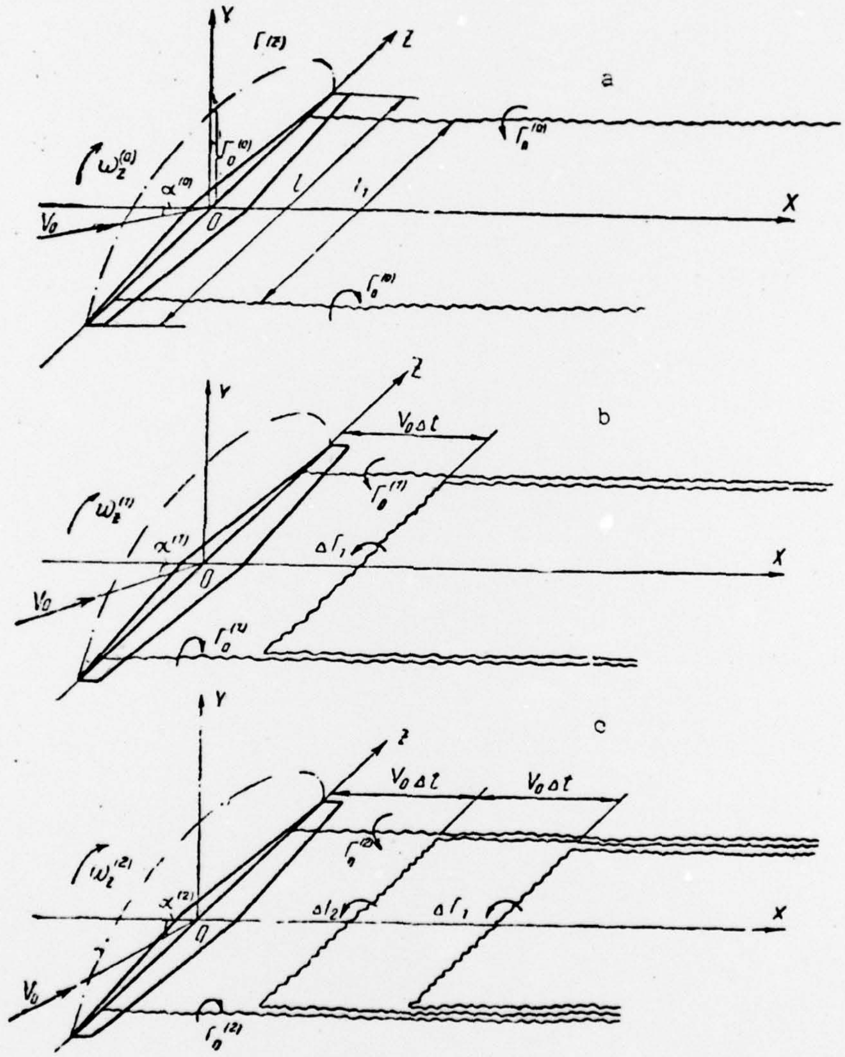

Figure 1.

With the selected law of wing span circulation distribution, the distance between free vortices depends solely on the geometrical characteristics of the wing. Actually, on the basis of (1.1), (1.3), and (1.4)

$$l_1 = 0.51\ (1.285 + 0.5k). \tag{1.5}$$

Circulation of the bound vortices is constant in accordance with the constant values of the angle of attack and angular velocity during steady motion. It changes in time during unsteady motion and this can be interpretted as the result of the formation of an additional rectilinear closed vortex line with length-wise uniform circulation $\Delta\Gamma$ (Figure 1, b). The transverse vortex with circulation - $\Delta\Gamma$ - is carried downstream and the distance from it to the bound vortices constantly increases, i.e., a free vortex of a new type - a transient free vortex - appears. Its downwash with a velocity V in the plane formed by the free steady vortices changes the circulation of the latter length-wise and in time.

In order to solve the problem in the case of an unsteady aperiodic motion, we shall present the continuous process of change of the aerodynamic characteristics of the wing in time in the form of a set of discrete changes. The kinematic parameter, and consequently, the aerodynamic characteristics change jump-wise at certain moments of time and the intervals between these moments are constant (Figure 2).

We choose the calculated moments of time (t_0, t_1, ..., t_n) such that they directly precede the moments in which the kinematic parameters change jump-wise, and thus, the circulation of the bound vortices. /9

At the beginning of unsteady motion (t_0), the circulation of vortices is determined by steady initial values of the kinematic parameters and the vortex system has the form shown in Figure 1, a. We shall call it a quasi-steady vortex system.

Inasmuch as t_0 is the beginning of unsteady motion, then there are no transient vortices at this calculation moment and

$$C_y^{(0)} = C_y^{\alpha}\alpha_a^{(0)} + C_y^{\bar{\omega}_z}\bar{\omega}_z^{(0)}. \tag{1.6}$$

where $\alpha_a^{(0)}$ and $\bar{\omega}_z^{(0)}$ are the initial values of the kinematic parameters which correspond to the beginning of unsteady motion; the parenthetical 0 as a superscript signifies the calculated moment of time when the value of the corresponding parameter is determined:

$$\alpha_a = \alpha - \alpha_0,$$

where α_0 is the zero lift angle;

$\frac{\omega_z b_A}{V}$ - non-dimensional pitch angular velocity (b_A - mid aerodynamic chord).

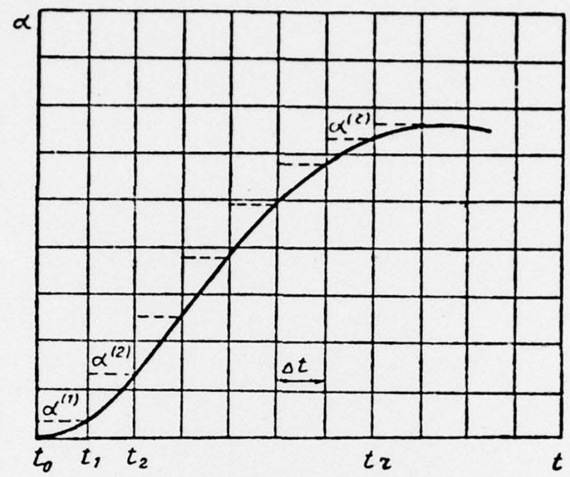

Figure 2.

Circulation of the bound vortices will change suddenly by a moment of time t_1, which will be accompanied by convergence of the transverse free vortex that will recede a distance $V\Delta t$ from the bound vortices by this time.

The lift factor at a calculated moment of time t_1 is

$$c_y^{(1)} = c_y^{\alpha} \alpha_a^{(1)} + c_y^{\bar{\omega}_z} \bar{\omega}_z^{(1)}; \tag{1.7}$$

where $\alpha_a^{(1)}$ and $\bar{\omega}_z^{(1)}$ are the values of the kinematic parameters after a single sudden change that corresponds to a moment of time t_1.

On the one hand, the velocity induced at any point can be viewed as the result of the effect of the quasi-steady vortex system that corresponds to a calculated moment of time t_0 and the additional closed cortex system shown by the dash line in Figure 1, b. But on the other hand, which is more convenient, one can consider that a new quasi-steady vortex system has formed at a calculated moment t_1 which has the previous law of distribution of circulation of the bound vortices along wing span given by expression (1.2) with a new value of the lift factor. /10

Two free vortex cores with uniform circulation $\Gamma_0^{(1)} = \Gamma_0^{(0)} + \Delta\Gamma_1$ run to infinity from the bound vortices, but then the transverse free circulation vortex - $\Delta\Gamma_1$ - also has longitudinal free vortex lines that also run to infinity, i.e., we obtain a new quasi-steady vortex system and one horseshoe vortex with a length (span) l_1 (Figure 1, c).

The magnitude of the circulation change $\Delta\Gamma_1$ can be calculated in the following way:

$$\Delta\Gamma_1 = \Gamma_0^{(1)} - \Gamma_0^{(0)} = [c_y^{(1)} - c_y^{(0)}]\frac{SV}{2l_1} = \Delta c_y \frac{SV}{2l_1}. \qquad (1.8)$$

The above is also valid when there is not one but a whole series of transient transverse free vortices behind the wing. At each succeeding calculated moment of time t_r, the horseshoe vortices examined at the previous moment of time $t_r - 1$ move downstream a distance $V\Delta t$ without a circulation change, and moreover, a new horseshoe vortex appears with a circulation $\Delta\Gamma_r$. The horseshoe vortex corresponds to the r-th change of the quasi-steady vortex system and will cover a distance $V\Delta t$ from the wing.

n changes of the quasi-steady vortex system occur and n systems of horseshoe vortices forms by a moment of time t_n. At a moment of time t_n, the kinematic parameters acquire values of $\alpha_a^{(n)}$ and $\omega_z^{(n)}$, and with this

$$c_y^{(n)} = c_y^{\alpha}\alpha_a^n + c_y^{\bar{\omega}_z}\bar{\omega}_z^{(n)} = c_y^{\alpha}\cdot\alpha_a^n + c_y^{\bar{\omega}_z}\cdot\bar{\omega}_z^n;$$

$$\Gamma_0^{(n)} = \frac{c_y^n SV}{l}\frac{1}{(1{,}285 + 0{,}5k)} = \frac{c_y^{(n)}SV}{2l_1};$$

$$\Delta c_{y_n} = c_y^{(n)} - c_y^{(n-1)} = c_y^{\alpha}\Delta\alpha_n + c_y^{\bar{\omega}_z}\Delta\bar{\omega}_{z_n} + c_y^{\omega_z}\cdot\Delta\bar{\omega}_{z_n};$$

$$\Delta\alpha_n = \alpha_a^{(n)} - \alpha_a^{(n-1)}; \quad \Delta\bar{\omega}_{z_n} = \bar{\omega}_z^{(n)} - \bar{\omega}_z^{(n-1)};$$

$$\Delta\Gamma_n = \Delta c_{y_n}\frac{SV}{2l_1}.$$

The given vortex model makes it possible to calculate the downwash angle behind the wing during unsteady aperiodic wing motion.

2. We shall solve a problem in the linear presentation, i.e., we shall consider the sheet of free vortices behind the wing to be plane and the free vortex cores, which represent the results of twisting of the vortex sheet, to be rectilinear and lying the the xOz plane. The linear presentation significantly simplifies the problem and does not result in large errors in the region of low angles of attack of the wing [5]. This region is of the greatest practical interest, for the low angles of attack of the wing are the critical ones for the stabilizer-elevator unit (in the sense of stall probability).

We shall consider that the wing moves translationally at a uniform velocity V_0, that flow braking behind the wing is absent and that the downwashes created by the wing are constant along the tail span. We determine the projections of velocity in the direction perpendicular to the free-stream flow (direction mm). These projections are induced by elements of the vortex model at calculation point A (ξ, η, 0), which is located in the plane of symmetry of the wing (Figure 3). /11

On the basis of a work 1 , the velocity induced by the system of bound vortices distributed along the wing span according to a law (1.2), is:

$$w_{10} = \frac{\Gamma_0}{2\pi l} \frac{1}{\sqrt{\xi^2+\eta^2}} \left[\mu + \frac{1}{\sqrt{1+\xi^2+\eta^2}} - (1-k)\sqrt{1+\xi^2+\eta^2} - \sqrt{\xi^2+\eta^2}\right].$$

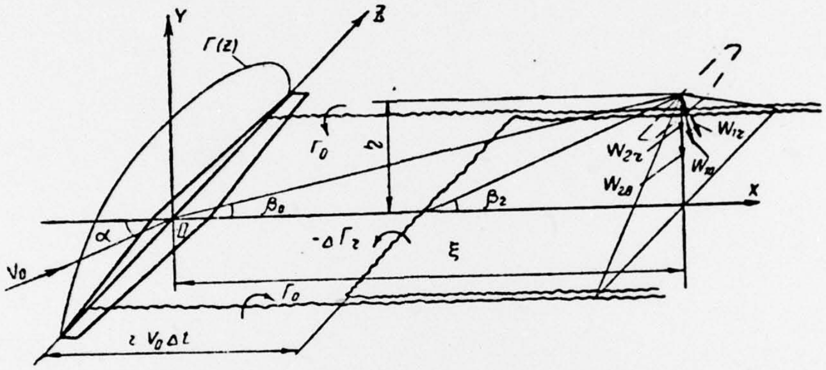

Figure 3.

Substituting (1.4) in this expression, we obtain

$$w_{10} = \frac{c_y V_0}{2\pi\lambda} \frac{1}{(1{,}285+0{,}5k)} \frac{1}{\sqrt{\xi^2+\eta^2}} \left[\mu + \frac{1}{\sqrt{1+\xi^2+\eta^2}} - \right. \tag{2.1}$$
$$\left. -(1-k)\sqrt{1+\xi^2+\eta^2} - \sqrt{\xi^2+\eta^2}\right],$$

where

$$\mu = \sqrt{1+\xi^2+\eta^2}\, \mathrm{E}\left(\frac{\pi}{2}, \frac{1}{\sqrt{1+\xi^2+\eta^2}}\right) - \frac{\xi^2+\eta^2}{\sqrt{1+\xi^2+\eta^2}} \mathrm{F}\left(\frac{\pi}{2}, \frac{1}{\sqrt{1+\xi^2+\eta^2}}\right).$$

μ is a function of the total elliptical integrals of the first and second orders.

The projection of velocity ω_{10} to direction mm is

$$\bar{w}_{10} = w_{10} \cdot \cos(\alpha - \beta_0), \tag{2.2}$$

where

$$\beta_0 = \text{arctg}\,\frac{\eta}{\xi}\,.$$

Angle β is nearly zero for aircraft whose stabilizer-elevator units are on the fuselage. In this case

$$\bar{w}_{10} = w_{10}\cos\alpha. \tag{2.3}$$

In the region of small angles of attack of the wing

$$\bar{\omega}_{10} = \omega_{10}. \tag{2.4}$$

Velocity induced by two vortex cores with circulation Γ_0 is

$$w_{20} = \frac{\Gamma_0}{\pi l}\frac{\nu}{\nu^2+\eta^2}\left(1+\frac{\xi}{\sqrt{\nu^2+\xi^2+\eta^2}}\right),$$

where

$$\nu = \frac{l_1}{l}.$$

Bearing in mind that $\Gamma_0 = \frac{c_y S V_0}{2\nu l}$, we express ω_{20} through coefficient Cy.

$$w_{20} = \frac{c_y V_0}{2\pi\lambda}\frac{1}{\nu^2+\eta^2}\left(1+\frac{\xi}{\sqrt{\nu^2+\xi^2+\eta^2}}\right). \tag{2.5}$$

The projection of velocity ω_{20} to direction mm is

$$\bar{\omega}_{20} = \omega_{20}\cos\alpha, \tag{2.6}$$

and in the region of small angles of attack is

$$\bar{\omega}_{20} = \omega_{20}. \tag{2.7}$$

Velocity induced by a free transverse vortex with circulation - $\Delta\Gamma_r$, is

$$w_{1r} = \frac{-\Delta\Gamma_r}{\pi l_1}\frac{1}{\sqrt{\xi_r^2+\eta^2}}\frac{\nu}{\sqrt{\nu^2+\xi_r^2+\eta^2}},$$

where

$$\xi_r = \xi - \frac{rV_0\Delta t}{\frac{l}{2}}.$$

Taking expression (1.8) into account, we write

$$w_{1r} = -\frac{\Delta c_{y_r} V_0}{2\pi\lambda\nu}\frac{1}{\sqrt{\xi^2+\eta^2}}\frac{1}{\sqrt{\nu^2+\xi_r^2+\eta^2}}. \tag{2.8}$$

The projection of velocity ω_{1r} to direction mm is

$$\bar{w}_{1r} = w_{1r}\cdot\cos(\alpha-\beta_r) \approx w_{1r}\cos\alpha\cdot\cos\beta_r, \tag{2.9}$$

where

$$\beta_r = \text{arctg}\,\frac{\eta}{\xi_r}.$$

In the region of small angles of attack

$$\bar{w}_{1r} = w_{1r} \cos \beta_r = w_{1r} \frac{\xi_r}{\sqrt{\xi_r^2 + \eta^2}}. \qquad (2.10)$$

Velocity induced by two longitudinal vortices with circulation - ΔΓr is

$$w_{2r} = -\frac{\Delta\Gamma_r}{\pi l} \frac{\nu}{\nu^2 + \eta^2} \left(1 + \frac{\xi_r}{\sqrt{\nu^2 + \xi_r^2 + \eta^2}}\right).$$

Substituting expression (1.8), we have

$$w_{2r} = -\frac{\Delta c_{y_r} V_0}{2\pi\lambda} \frac{1}{\nu^2 + \eta^2} \left(1 + \frac{\xi_r}{\sqrt{\nu^2 + \xi_r^2 + \eta^2}}\right). \qquad (2.11)$$

The projection of velocity ω_{2r} to direction mm

(2.12)

The region of small angles of attack

$$\bar{w}_{2r} = w_{2r} \cos \alpha. \qquad (2.13)$$

Rake angle at a calculation point A is

$$\bar{w}_{2r} = w_{2r}. \qquad (2.14)$$

because of the effect of the vortex systems examined above, where

$$\chi_0 = \frac{1}{(1{,}285 + 0{,}5k)\sqrt{\xi^2 + \eta^2}} \left[\mu + \frac{1}{\sqrt{1 + \xi^2 + \eta^2}} - (1 - k)\sqrt{1 + \xi^2 + \eta^2} - \right.$$
$$\left. - \sqrt{\xi^2 + \eta^2}\right] + \frac{1}{\nu^2 + \eta^2}\left(1 + \frac{\xi}{\sqrt{\nu^2 + \xi^2 + \eta^2}}\right); \qquad (2.15)$$
$$\chi_r = \left[\frac{\xi_r}{\nu(\xi_r^2 + \eta^2)\sqrt{\nu^2 + \xi_r^2 + \eta^2}}\right] + \frac{1}{\nu^2 + \eta^2}\left[1 + \frac{\xi_r}{\sqrt{\nu^2 + \xi_r^2 + \eta^2}}\right]. \qquad (2.16)$$

In the region of small angles of attack

$$\varepsilon_{\text{кр}} = \frac{1}{2\pi\lambda}(c_y \chi_0 - \Delta c_{y_r} \chi_r). \qquad (2.17)$$

Value χ_0 is solely a function of the coordinates of the calculated point and does not depend on time, and χ_r is a function of the coordinates of the calculated point and value ξ_r. With assigned wing velocity and a chosen span of time Δt, value ξ_r changes according to a linear law that multiply corresponds to calculated moment r (r = 0, 1, 2, ..., n).

Taking the above into account, it is easy to devise a method of calculating the downwash angle behind a wing in the region of the stabilizer-elevator unit during unsteady aperiodic motion. Actually, by successively examining the corresponding vortex models, we calculate the projection of velocity induced by all bound and free vortices at each calculated moment of time to direction mm, and then also calculate the corresponding downwash angle.

The vortex model appears as shown in Figure 1, a at a calculated moment of time t_0 ($r = 0$) at the beginning of unsteady motion, i.e., it consists solely of a quasi-steady vortex system. Designating the calculated moment of time with the index in parentheses and taking into account that $\Delta C_{yr} = 0$ when $r = 0$, one can write the following on the basis of (2.14)

$$\varepsilon_{cr}^{(0)} = \frac{\bar{\omega}_{10}^{(0)} + \bar{\omega}_{20}^{(0)}}{V_0} = \frac{c_y^{(0)}}{2\pi\lambda} \chi_0 \cos\alpha,$$

where $C_y^{(0)}$ is calculated for values of the kinematic parameters $\alpha_a^{(0)}$ and $\bar{\omega}_{20}^{(1)}$, and χ_1 are calculated according to formula (2.16) when $r = 1$;

$$\Delta c_{y_1} = c_y^1 - c_y^{(0)}.$$

The vortex model consists of a quasi-steady vortex system and horseshoe vortices whose number is equal to p at a calculated moment of time t_p.

Downwash angle /14

$$\begin{aligned} \varepsilon_{cr}^{(p)} &= \frac{\bar{\omega}_{10}^{(p)} + \bar{\omega}_{20}^{(p)} + \bar{\omega}_{11}^{(p)} + \bar{\omega}_{21}^{(p)} + \ldots + \bar{\omega}_{1k}^{(p)} + \bar{\omega}_{2k}^{(p)} + \ldots + \bar{\omega}_{1p}^{(p)} + \bar{\omega}_{2p}^{(p)}}{V_0} = \\ &= \frac{\cos\alpha}{2\pi\lambda} \left[c_y^{(p)}\chi_0 - \Delta c_{y_1}\chi_p - \Delta c_{y_2}\chi_{p-1} - \ldots - \Delta c_{y_k}\chi_{p-(k-1)} - \ldots - \Delta c_{y_p}\chi_1\right] = \\ &= \frac{\cos\alpha}{2\pi\lambda}\left[c_y^{(p)}\chi_0 - \sum_{i=1}^{p}\sum_{j=p}^{1}\Delta c_{y_i}\chi_j\right]. \end{aligned} \qquad (2.17)$$

In the region of small angles of attack of the wing

$$\varepsilon_{cr}^{(p)} = \frac{1}{2\pi\lambda}\left[c_y^{(p)}\chi_0 - \sum_{i=1}^{p}\sum_{j=p}^{1}\Delta c_{y_i}\chi_j\right], \qquad (2.18)$$

where

$$\begin{aligned} c_y^{(p)} &= f\left[\alpha_a^{(p)},\ \bar{\omega}_z^{(p)}\right]; \\ \Delta c_{y_1} &= c_y^{(1)} - c_y^{(0)}; \\ &\ldots\ldots\ldots\ldots \\ \Delta c_{y_p} &= c_y^{(p)} - c_y^{(p-1)}. \end{aligned}$$

The values $\chi_1, \chi_2, \ldots, \chi_p$ are calculated according to formula (2.16) with the corresponding values $r = 1, 2, \ldots, p$.

3. At high subsonic flight velocities, one can calculate downwash angles behind the wing using the Prandtl-Glauert transform.

We transform the coordinate system such that

$$x_M = \frac{x}{\sqrt{1 - M_\infty^2}};\ y_M = y;\ z_M = z, \qquad (3.1)$$

where x_M, y_M and z_M are the coordinates of a point in transformed space which corresponds to the calculated one.

The geometrical parameters of the wing in the transformed space are:

$$\lambda_M = \lambda\sqrt{1-M_\infty^2};\ c_M = c;\ l_M = l;\ b_M = \frac{b}{\sqrt{1-M_\infty^2}};\ S_M = \frac{S}{\sqrt{1-M_\infty^2}}. \qquad (3.2)$$

We shall take the following kinematic parameters for the transformed wing:

$$V_{0M} = V_0;\ \alpha_M = \alpha;\ \bar{\omega}_{zM} = \bar{\omega}_z,$$

(here $\omega_{zM} = \omega_z\sqrt{1-M_\infty^2}$), then the inductive velocities in the corresponding points of the original and transformed spaces are linked by relationships [2]

$$w_x = \frac{w_{xM}}{\sqrt{1-M_\infty^2}};\ w_y = w_{yM};\ w_z = w_{zM}. \qquad (3.3)$$

and the following relationships exist

$$c_{yM}^{\alpha} = c_y^{\alpha}\sqrt{1-M_\infty^2};\ c_{yM}^{\bar{\omega}_z} = c_y^{\bar{\omega}_z}\sqrt{1-M_\infty^2};\ c_{yM} = c_{yM}^{\alpha}\alpha_a + c_{yM}^{\bar{\omega}_z}\bar{\omega}_z. \qquad (3.4)$$

We calculate the parameters of the transformed wing and the coordinates of the corresponding point according to the assigned geometrical parameters of the wing and the coordinates of the calculated point according to formulas (3.2) and (3.1). Then we determine the inductive velocities in transformed space on the basis of expressions (2.1), (2.5), (2.8), and (2.11). Knowing the inductive velocities according to formulas (3.3) and (2.17), we find the downwash angles behind the actual wing at the given Mach number.

$$\varepsilon_{cr\,\text{cж}}^{(p)} = \frac{\cos\alpha}{2\pi\lambda_M}\left[c_{yM}^{(p)}\chi_{0M} - \sum_{i=1}^{p}\sum_{j=p}^{1}\Delta c_{yM_i}\chi_{M_j}\right] = \qquad (3.5)$$

$$= \frac{\cos\alpha}{2\pi\lambda}\left[c_y^{(p)}\chi_{0M} - \sum_{i=1}^{p}\sum_{j=p}^{1}\Delta c_{y_i}\chi_{M_j}\right].$$

4. The angle of attack of the stabilizer-elevator unit in steady curvilinear flight is determined by the following relationship:

$$\alpha_{s.c.} = \alpha + \phi - \varepsilon_{cr} - \varepsilon_0 + \Delta\alpha_{s.c.}, \qquad (4.1)$$

where α is the angle of attack of the wing;

ϕ is the rigging angle of incidence of the stabilizer-elevator unit;

ε_{cr} is the downwash angle caused by the wing;

ε_0 is the downwash angle when $C_y = 0$.

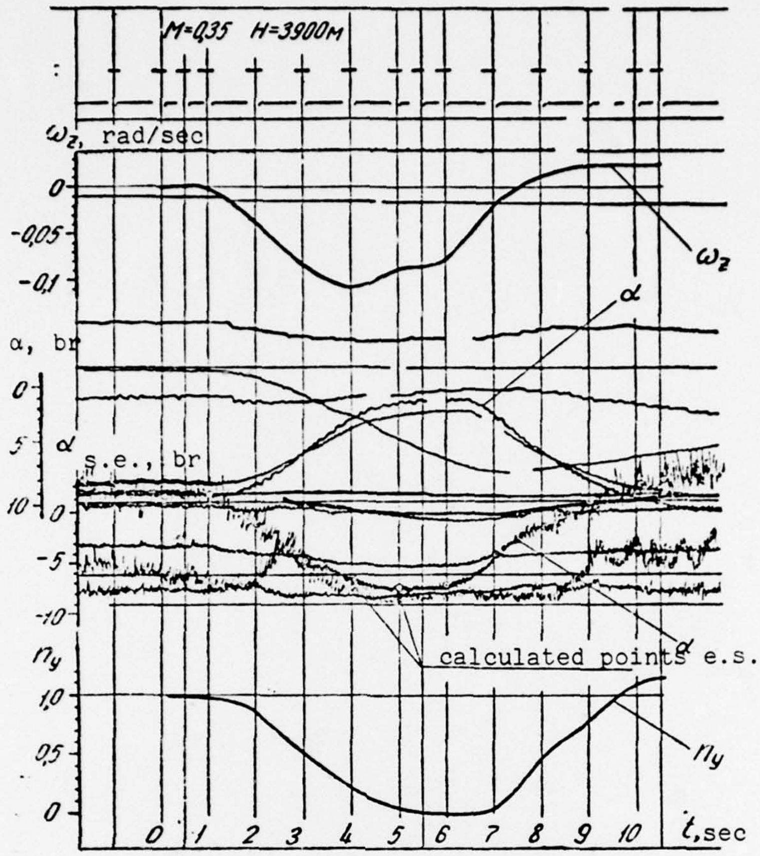

Figure 4.

$$\Delta\alpha_{r.o} = \frac{\omega_z}{V_0}\frac{l}{2}\xi - \qquad (4.2)$$

- the additional change of the angle of attack of the stabilizer-elevator unit caused by turning of an aircraft with angular velocity ω_z.

One can calculate the angle of attack of the stabilizer-elevator unit by the numerical method during unsteady aperiodic motion in the vertical plane /16 using formulas (4.1) and (4.2), having replaced the continuous processes with discrete ones. Successively examining the vortex models that correspond to calculated moments of time t_0, t_1, ..., t_p, we determine the downwash angles $\varepsilon_{cr}^{(0)}$, $\varepsilon_{cr}^{(1)}$, ..., $\varepsilon_{cr}^{(p)}$ and the additional changes of the angle of attack of the stabilizer-elevator unit $\Delta\alpha_{s.e}^{0}$, $\Delta\alpha_{s.e}^{(1)}$, $\Delta\alpha_{s.e}^{(p)}$, according to formula (2.17).

The angle of attack of the stabilizer-elevator unit at a calculated moment of time t_o is

$$\alpha^{(p)}_{s.e} = \alpha^{(p)} + \varphi - \varepsilon^{(p)}_{cr} - \varepsilon_0 + \Delta\alpha^{(p)}_{s.e},$$

where

$$\alpha^{(p)} = \alpha^{(p)}_{a} - \alpha_0;$$

$$\Delta\alpha^{(p)}_{s.e} = \frac{\omega_z^{(p)}}{V_0}\,\frac{l}{2}\,\xi.$$

Figures 4 - 6 show oscillograms with recordings of the kinematic parameters of aircraft motion when the aircraft is maneuvering in the vertical plane ("giving it" the elevator). The operating conditions of the engines are flight idling (B = 0); flaps up.

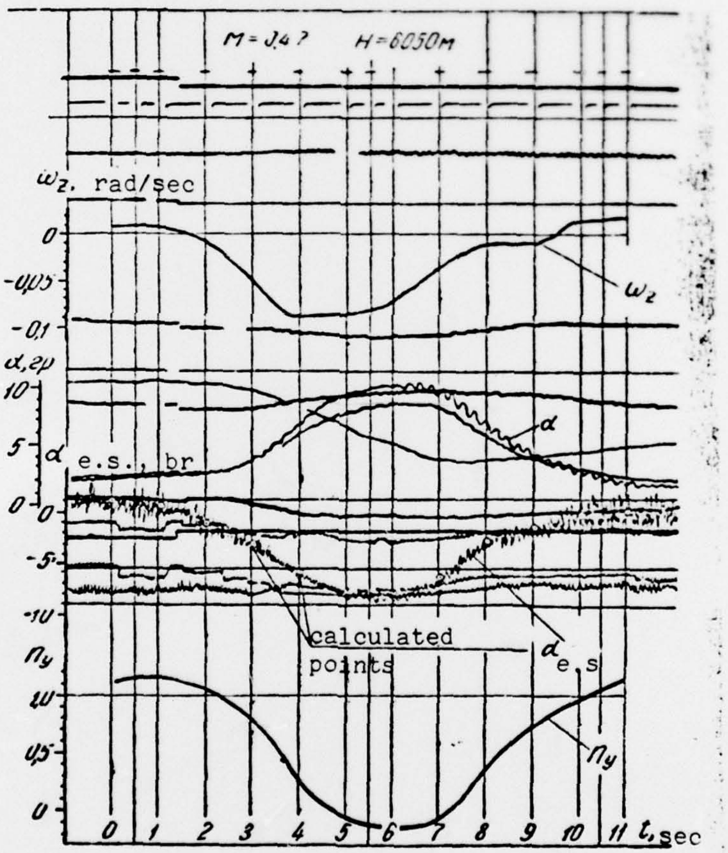

Figure 5.

The coordinates of the calculated point are: $\xi = 0.697$; $\eta = 0.093$; $\zeta = 0$. The wing parameters are: $\lambda = 12$; $k = 0.384$; $\alpha_0 = 1^\circ$.

The results of calculations made according to the suggested method are in good agreement with the data of a flight experiment carried out in broad ranges of angle of attack and Mach number.

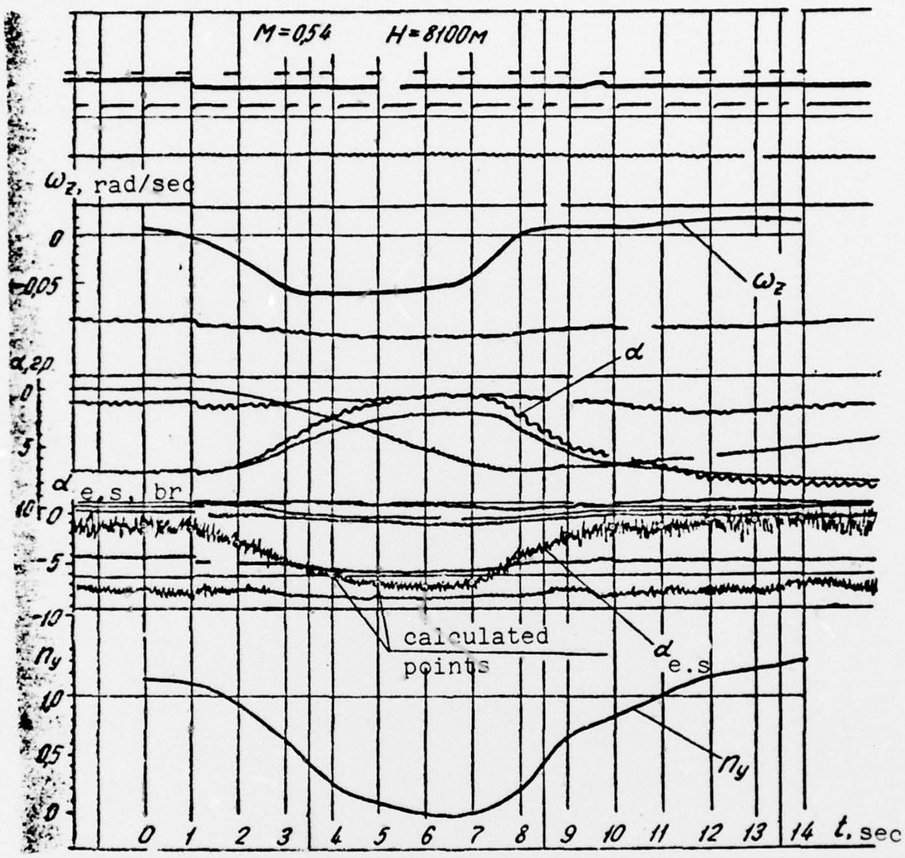

Figure 6.

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